DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

316. Proposed by B. F. FINKEL, Ph. D.

Prove that $\sum\limits_{r=1}^{r=n}(-1)^{n-1}\frac{1}{n}{}_nC_r=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}$ where ${}_nC_r=\frac{n(n-1)\ldots.(n-r+1)}{1.2.3\ldots r}$. Dickson's *College Algebra*, ex. 13, p. 92.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

The first member of the equation should be $\sum\limits_{r=1}^{r=n}(-1)^{r-1}\frac{1}{r}{}_nC_r$ instead of $\sum\limits_{r=1}^{r=n}(-1)^{n-1}\frac{1}{n}{}_nC_r$.

$$S_n=\sum_{r=1}^{r=n}(-1)^{r-1}\frac{1}{r}{}_nC_r=n-\frac{n(n-1)}{2.2!}+\frac{n(n-1)(n-2)}{3.3!}-\text{ to } n \text{ terms.}$$

$$S_{n+1}=(n+1)-\frac{(n+1)n}{2.2!}+\frac{(n+1)(n-1)}{3.3!}-\text{ to } n+1 \text{ terms.}$$

$$\therefore S_{n+1}-S_n=1-\frac{n}{2\,!}+\frac{n(n-1)}{3\,!}-\text{ to } n+1 \text{ terms}=\frac{1}{n+1}[1-(1-1)^{n+1}]=\frac{1}{n+1}.$$

$$S_1=1,\ S_2-S_1=\tfrac{1}{2} \text{ or } S_2=1+\tfrac{1}{2},$$
$$S_3-S_2=\tfrac{1}{3} \text{ or } S_3=1+\tfrac{1}{2}+\tfrac{1}{3},$$
$$. \quad . \quad . \quad . \quad . \quad . \quad .$$
$$S_n-S_{n-1}=1/n \text{ or } S_n=1+\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}+\ldots+1/n.$$

(Vol. VII, No. 4, of the MONTHLY, page 105, line 8, gives $S_{n+1}-S_n=\frac{1}{1+n}$ by substituting -1 for x.)

Also solved by S. Lefsehetz.

317. Proposed by FRANCIS RUST, Allegheny, Pa.

Once, in classic days, Silenus lay asleep; a goat skin filled with wine near him. Dionysius passing by, profited, by siezing the skin, and drinking for two-thirds ($\frac{2}{3}$) of that time in which Silenus alone could have emptied said skin. At this point Silenus awoke, and seeing what was happening, snatched away the precious skin, and finished it.

Now, had both started together, and drank simultaneously, they would have consumed the wine skin in two hours less time. And, in this case, Dionysius' share would have been one-half as much as Silenus did secure, by waking and snatching the skin.

In what time would either one of them alone finish the goat-skin?

Solution by PROFESSOR F. L. GRIFFIN, Ph. D., Williams College.

Let x=fractional part which S drank, and y=number of hours S requires for entire skin. Then $\frac{2}{3}y$=time D was drinking, and xy=time S was drinking; $y(\frac{2}{3}+x)$ = time they used consecutively. Also, since $\frac{3(1-x)}{2y}$=part D drinks per hour, or $\frac{3(1-x)+2}{2y}$=part both drink per hour, the time required when drinking simultaneously$=\frac{2y}{5-3x}$.

Hence, (A) $y(\frac{2}{3}+x)=2+\frac{2y}{5-3x}$.

Again, the part D would get when they drink simultaneously $=\left(\frac{2y}{5-3x}\right)\frac{3(1-x)}{2y}$, or $\frac{3-3x}{5-3x}$; hence, by the problem, (B) $\frac{3-3x}{5-3x}=\frac{1}{2}x$.

Equation (B) gives $x=\frac{2}{3}$ or 3, the latter value being impossible.

Then (A) becomes $\frac{4}{3}y=2+\frac{2}{3}y$, or y=3; and since D drinks $\frac{1}{6}$ per hour, his time would be 6 hours.

Also solved by V. M. Spunar, G. B. M. Zerr, and J. Scheffer.

318. Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.

Sum to infinity the series $n/(4n^2-1)^2$ beginning with $n=1$.

Solution by J. W. CLAWSON, Ursinus College, Collegeville, Pa.; HOWARD C. FEEMSTER, A. B., York College, York, Neb.; J. EDWARD SANDERS, Weather Bureau, Chicago, Ill., and S. LEFSEHETZ, Pittsburg, Pa.

$$\sum_{n=1}^{n=\infty}\frac{n}{(4n^2-1)^2}=\sum_{n=1}^{n=\infty}\tfrac{1}{8}\left[\frac{1}{(2n-1)^2}-\frac{1}{(2n+1)^2}\right]$$

$$=\underset{n=\infty}{\text{Lt.}}\tfrac{1}{8}\left[\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots\frac{1}{(2n-1)^2}\right)\right.$$

$$\left.-\left(\frac{1}{3^2}+\frac{1}{5^2}+\dots\frac{1}{(2n-1)^2}+\frac{1}{(2n-1)^2}\right)\right]=\underset{n=\infty}{\text{Lt.}}\tfrac{1}{8}\left[\frac{1}{1^2}-\frac{1}{(2n+1)^2}\right]=\tfrac{1}{8}.$$

Also solved by V. M. Spunar, G. B. M. Zerr, J. Scheffer, S. A. Corey, and T. J. Fitzpatrick.

GEOMETRY.

343. Proposed by O. J. BROWN, Fairhope, Ala.

From any external point of a triangle, to draw a line so as to divide the triangle into two equal parts.